E1:

Please show that Polynomial space (<= 2) is isomorphic to i-j-k space that you have learned in Calculus.

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We know that P2 is a vector space with standard basis B = {1, x, x2}

Consider the coordinate mapping x |-> [x]B . Let u and v be 2 typical vectors from P2.

We know u = c1\*t2 + c2\*t + c3\*1, v = d1\*t2 + d2\*t + d3\*1

So B-coordinate vectors of u and v are the following.

[u]B = (c1, c2, c3) and [v]B = (d1, d2, d3)

u + v = (c1+d1)\*t2 + (c2+d2)\*t + (c3+d3)\*1, So [u]B + [v]B = [u+v]B

Let r be real number scaler,

r\*u = (r\*c1)\*t2 + (r\*c2)\*t + (r\*c3)\*1, So [r\*u]B = r\*[u]B.

The above shows that this transformation is linear. Also since B is a basis of P2 , this linear transformation is 1-1.

We can conclude that the linear transformation form P2 to i-j-k space is a isomorphic.

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E2:

Please define Polynomial space Pn (<= n ) for any non-negative integer

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To show Pn a vector space is to show that Pn fulfill that 8 axioms:

1> Associativity:

(a1tn + a2t(n-1) … +an\*1) + [(b1tn + b2t(n-1) … +bn\*1) + (c1tn + c2t(n-1) … +cn\*1)] =

[(a1tn + a2t(n-1) … +an\*1) + (b1tn + b2t(n-1) … +bn\*1)] + (c1tn + c2t(n-1) … +cn\*1) =

(a1+b1+c1)tn + (a2+b2+c2)t(n-1) … +(an+bn+cn)\*1

2> Commutativity:

let a = (a1tn + a2t(n-1) … +an\*1), and b = (b1tn + b2t(n-1) … +bn\*1), a and b in Pn

a + b = b + a = (a1+b1)tn + (a2+b2)t(n-1) … +(an+bn)\*1)

3> Additive Identity:

let a = (a1tn + a2t(n-1) … +an\*1), a in Pn

a + 0 = (a1+0)tn + (a2+0)t(n-1) … +(an+0)\*1) = a

4> Addition Inverse:

let a = (a1tn + a2t(n-1) … +an\*1), a in Pn

a + (-a) =(a1-a1)tn + (a2-a2)t(n-1) … +(an-an)\*1 = 0tn + 0t(n-1) … +0\*1 = 0

5> Scalar multiplication:

let a = (a1tn + a2t(n-1) … +an\*1), a in Pn . let c1, c2 be scalers

c1\*(c2\*a) = c1\*((a1\*c2)tn + (a2\*c2)t(n-1) … +(an\*c2)\*1) =(a1\*c1\*c2)tn + (a2\*c1\*c2)t(n-1) … +(an\*c1\*c2)\*1

(c1\*c2)\*a = (a1\*c1\*c2)tn + (a2\*c1\*c2)t(n-1) … +(an\*c1\*c2)\*1

So c1\*(c2\*a) = (c1\*c2)\*a

6> Multiplicative Identity:

let a = (a1tn + a2t(n-1) … +an\*1), a in Pn .

1\*a = (1\*a1)tn + (1\*a2)t(n-1) … +(1\*an)\*1

7> Distributive Property:  
let a be a scaler

let b = (b1tn + b2t(n-1) … +bn\*1), b in Pn .

let c = (c1tn + c2t(n-1) … +cn\*1), c in Pn .

a\*(b+c) = a\*((b1+c1)tn + (b2+c2)t(n-1) … +(bn+cn)\*1) = (a\*b1+a\*c1)tn + (a\*b2+a\*c2)t(n-1) … +(a\*bn+a\*cn)\*1

ab + ac = (a\*b1)tn + (a\*b2)t(n-1) … +(a\*bn)\*1 + (a\*c1)tn + (a\*c2)t(n-1) … +(a\*cn)\*1 = (a\*b1+a\*c1)tn + (a\*b2+a\*c2)t(n-1) … +(a\*bn+a\*cn)\*1

So a\*(b+c) = ab + ac

8> Distributive property for scalars:

let a be a scaler

let b be a scaler

let c = (c1tn + c2t(n-1) … +cn\*1), c in Pn .

(a+b)\*c = (a+b)\*c1tn + (a+b)\*c2t(n-1) … +(a+b)\*cn\*1

a\*c + b\*c = a\*c1tn + a\*c2t(n-1) … +a\*cn\*1 +b\*c1tn +b\*c2t(n-1) … +b\*cn\*1 = (a+b)\*c1tn + (a+b)\*c2t(n-1) … +(a+b)\*cn\*1

So (a+b)\*c = a\*c + b\*c

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E3:

Is the total union, denoted by P\*,  of all Pn a vector space?

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The basis for Pn is B = {1, t, t2, …, tn}

Consider P2 , it has basis B = {1, t, t2 }. Is not hard to see that all Pn’ (n’ <= n) is a subset of Pn . So P\* should be a subset of Pn . Since Pn is a vector space, P\*, composed by all Pn , should also be a vector space.

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E5:

Find the relationships between P\* and P¥   
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Since all elements in Pinfinit can be generated by

, which is actually equivalent to Pn. Since P\* is a subset of Pn , P\* is a subset of Pinfinit.

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E6:

Have you learned the concept of the convergence of a sequence?

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Yes, I did in AP calculus BC.